Physics IV
ISI B.Math
Midterm Exam : February 20,2017

Total Marks: 60
Time : 3 hours
Answer all questions

1. (**Marks: 2 + 2 + 2 + 2 + 2 + 2 = 12**)

Select the correct option :

(i) Two clocks at the extreme ends of a train are synchronized with respect to the train frame. You are standing on the ground and the train moves past you. To you, compared to a clock at the back of the train, a clock at the front of the train will show
(a) the same time
(b) a later time
(c) an earlier time

(ii) It is given that the x component of a four-vector is zero in every inertial reference frame. By making appropriate coordinate transformations, i.e. any combination of rotations and Lorentz transformations, it is possible to transform it to the following form in some reference frame.$a, b, c \neq 0$
(a) $(a, 0, 0, 0)$
(b) $(0, 0, b, c)$
(c) $(0, 0, 0, 0)$

(iii) A heavy particle at rest decays into two lighter particles. The mass of the heavy particle must be
(a) greater than the sum of the masses of the decay products.
(b) equal to the sum of the masses of the decay products.
(c) less than the sum of the masses of the decay products.

(iv) An observer in Delhi reports a bomb explosion at 12 pm and another observer reports a bomb explosion in Kolkata at 2pm. Which of the following statements is true ?
(a) It is possible to find a frame of reference in which the two bomb explosions occur simultaneously.
(b) All inertial observers will agree that the time interval between the explosions is 2 hours.
(c) It is not possible to find a frame of reference in which the Kolkata explosion happens before the Delhi explosion.

(v) In relativistic mechanics, acceleration is in the direction of the applied force
(a) only if the force is parallel to the velocity of the particle
(b) only if the force is perpendicular to the the velocity of the particle
(c) never

(vi) Which of the following four vectors does not represent a null vector ?
(a) (1,0,1,0)
(b) (0,1,1,0)
(c) (1,1,0,0)

2. (**Marks : 1 + 1 + 2 + 1 + 1 + 1 + 1 + 4 = 12**)

Draw the ct and x axis of the spacetime coordinates of the observer $\mathcal{O}$. Then draw

(a) the world line of $\mathcal{O}$'s clock at $x = 1m$.

(b) ct' and x' axes of an observer $\mathcal{O}'$ who moves with a velocity $v = 0.5c$ in a positive x direction relative to $\mathcal{O}$ and whose origin ($x' = ct' = 0$) coincides with that of $\mathcal{O}$.

(c) the locus of events whose spacetime interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$ from the origin = - 1 m^2. At what coordinates (x', ct') does this curve intersect the ct' axis ?

(d) any two events that are timelike separated according to observer $\mathcal{O}$. Will these two events be timelike separated according to $\mathcal{O}$ as well ? Explain .

(e) the locus of events whose spacetime interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$ from the origin = 0.

(f) the locus of events, all of which occur simultaneously at $ct = 2m$ according to $\mathcal{O}$.

(g) the locus of events, all of which occur simultaneously at $ct' = 2m$ according to $\mathcal{O}'$.

(h) Show that the ratio of one x' unit to one x unit is $\sqrt{\frac{5}{3}}$

3. (**Marks : 3 + 3 + 3 + 3 = 12**)

(a) Suppose that a four vector $X \neq 0$ is timelike or null. If $X^0 > 0$ in some inertial coordinate system, then $X^0 > 0$ in every inertial coordinate system.

(b)Show that every four-vector orthogonal to a timelike vector is spacelike. (Two four vectors A^μ and B^μ are orthogonal if $A^\mu B_\mu = B^\mu A_\mu = 0$ $\mu = 0, 1, 2, 3$)

(c) If P^μ is the four-momentum of a particle and F^μ is the four-force defined as the product of the mass and the four-acceleration , show that $P^\mu F_\mu = 0$.

(d) Using the result in (c) , show that the four force F can be written as $F = (\frac{\gamma}{c}\mathbf{f}\cdot\mathbf{v}, \gamma\mathbf{f})$ where $\mathbf{v}$ is the three-velocity of the particle and $\mathbf{f}$, the three force $= \frac{d\mathbf{p}}{dt}(\mathbf{p} = m\gamma\mathbf{v}$. $\mathbf{p}$ being the relativistic three momentum).

4. (**Marks : 2 + 3 + 2 + 3 + 2 = 12**)

A train of proper length L moves at speed $3c/5$ with respect to the ground. A passenger runs from the back of the train to the front with a speed $c/2$ with respect to the train. What is the distance covered and the time taken by the passenger

(i) In the train frame ?

(ii) In the ground frame ?

(iii) In a frame of the passenger ?

(iv) Show that the invariant interval is the same in all three cases.

(v) Show that the times in the passenger frame and ground frame are related by the relevant γ factor

5. (**Marks : 6 + 6 = 12**)

(a) A particle of mass m_1 and velocity $\mathbf{v_1}$ collides with a stationary particle of mass m_2 and is absorbed by it. Find the mass m and the velocity $\mathbf{v}$ of the compound system.

(b) A photon with energy E collides with a mass M. The mass M scatters at an angle. If the resulting photon moves perpendicular to the incident photon's direction, what is its energy ?